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ON POLLUTION DISTRIBUTION ON UNHAS LAKE USING TWO DIMENSION ADVECTION-DIFFUSION EQUATION

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Abstract

The idea of this paper is to employ a simple numerical method to solve the pollution distribution and transportation problems around us. Numerical schemes such as forward, backward, centered difference are mixed by considering the domain and position of the boundary. Idea and schemes are applied into the Unhas lake with irregular shape of the boundary. Real pollutants data such as the fecal coliform and the total suspended solid content are supplied as the boundary condition for the steady state problems. Numerical results are used to describe the distribution of these pollutants for practical implications.

1. Introduction

The study of shallow water pollution distributions is considered

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important in Indonesia. Pollution in reservoir and river affect many living dimensions of society nearby. Therefore, transportation and pollution distributions in the shallow water become an interesting subject to be studied. Numerous authors studied pollution transportation and distributions through advection-diffusion equation. One dimension advection diffusion of plankton distribution in a coarse resolution of the ocean model can be found in [1]. A numerical algorithm for solving advection-diffusion equation with constant and variable coefficients especially for simple geometry boundary can be found in [2]. Recently, two dimension advection-diffusion equation involving real irregular domain of interest such as Unhas lake using Du-Fort Frankel method had been studied by [3]. Also, a generalized finite difference method to solve the three dimension advection-diffusion equation is found in [4].

However, not many studies have been done to solve the problems around our practical daily activities. Recently, a study on water quality analysis using physical parameter analysis, total suspended solid (TSS), total dissolved solid (TDS), chemical parameter analysis and microbiological parameter analysis in order to compare Unhas lake water and the standard government regulation got carried out in [5]. Collecting and analyzing such samples becomes expensive and restricted to the sample position. In this paper, we estimate the pollution distribution such as coliform bacteria and the total suspended solid (TSS) all over the Unhas lake using the compound finite difference schemes.

This paper is organized as follows. In Section 2, we introduce the basic equation and the problems. In Section 3, the grid computation, the code and the finite difference schemes are introduced. Section 4 presents some numerical results and discussions. Finally, conclusion can be found in Section 5.

2. Basic Equation and Problems

The model of pollutant distribution in Unhas lake satisfies the basic equation for two dimension advection diffusion problem [2-4, 6, 7] is

$$\frac{\partial C}{\partial t} + V_x \frac{\partial C}{\partial x} + V_y \frac{\partial C}{\partial y} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2}, \quad t > 0, \quad (x, y) \in \Omega \tag{1}$$

with initial condition

$$C(x, y, 0) = f(x, y) \tag{2}$$

and the boundary condition

$$aC(x, y, t) + b\frac{\partial C(x, y, t)}{\partial n} = g(t), \quad (x, y) \in \Gamma, \tag{3}$$

where C represents concentration of pollutant, V_x, V_y represent velocity constants with respect to x and y directions, respectively, D_x, D_y represent diffusion coefficients, $f(x, y)$ and $g(t)$ are known functions, a, b are constants and Γ is the boundary of Ω.

However, from the practical point of view, it is very difficult to obtain the complete solution for the whole problem. Initial condition and the boundary condition are affected dynamically by water intake and outtake from the lake. During the rainy season, the lake is flooded, water flow becomes irregular, and the turbulence contains materials of different kinds. During dry season, the lake is relatively calm and has a limited or no water flow. The pollutant concentration reaches its equilibrium state. For the case when the concentration is in the equilibrium state and there is no change with respect to time or

$$\frac{\partial C}{\partial t} = 0, \tag{4}$$

the model or problem simply reduces to

$$V_x \frac{\partial C}{\partial x} + V_y \frac{\partial C}{\partial y} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2}, \quad (x, y) \in \Omega \tag{5}$$

and the boundary condition becomes

$$aC(x, y) + b\frac{\partial C(x, y)}{\partial n} = g, \quad (x, y) \in \Gamma. \tag{6}$$

3. Grid Computation, Code and Finite Difference Schemes

As the domain of interest in studying pollution distribution of shallow water, we consider the Unhas lake. Unhas lake is located in the campus of Hasanuddin University, Tamalanrea, 9km. Makassar. This spreads approximately from $119^\circ 29' 22'' E$ and $5^\circ 8' 13.8'' S$ up to $119^\circ 29' 26.8'' E$ and $5^\circ 8' 6.7'' S$ as given by [5]. In physical measurement, it spreads 400m long and 320m wide as given by the picture taken from Google map in Figure 1. This domain area is then divided into 40 equal space grids long and 32 grids wide or 10 square meters each. These computation grid points are being coded from 0 up to 9, see [3]. The code marks from 0 up to 9 mean exterior point, interior point, lower side point, right side point, upper side point, left side point, lower left point, lower right point, upper left point, upper right point, respectively. A compound standard finite difference scheme is used to approximate advection-diffusion equation with respect to the codes. The grid point with code 1 means interior point which is approached by center difference scheme for both x and y. The grid point with code 3 means right side point which is approached by backward difference for x and center difference for y. The grid point with code mark 6 means lower left point which is approached by forward difference for x and y. The choice of the codes depends on the grid of the domain. The complete codes and finite difference approximation schemes are found in Table 1.

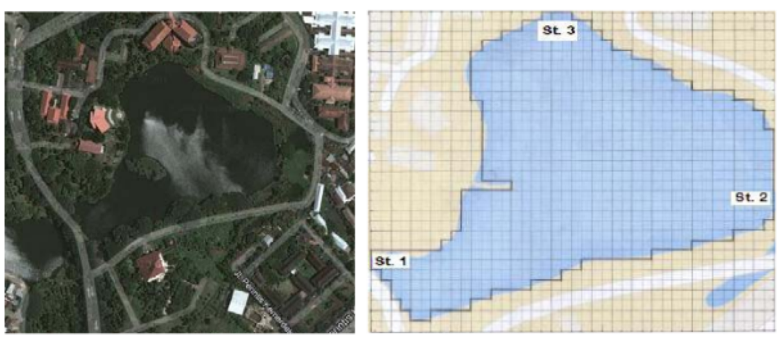

Figure 1. Unhas lake and grid computation.

Table 1. Codes and finite difference schemes

Code	Finite difference schemes
1	$V_x\left(\frac{C_{i+1j}-C_{i-1j}}{2\Delta x}\right)+V_y\left(\frac{C_{ij+1}-C_{ij-1}}{2\Delta y}\right)=D_x\left(\frac{C_{i+1j}-2C_{ij}+C_{i-1j}}{\Delta x^2}\right)+D_y\left(\frac{C_{ij+1}-2C_{ij}+C_{ij-1}}{\Delta y^2}\right)$
2	$V_x\left(\frac{C_{i+1j}-C_{i-1j}}{2\Delta x}\right)+V_y\left(\frac{C_{ij+1}-C_{ij}}{\Delta y}\right)=D_x\left(\frac{C_{i+1j}-2C_{ij}+C_{i-1j}}{\Delta x^2}\right)+D_y\left(\frac{C_{ij+2}-2C_{ij+1}+C_{ij}}{\Delta y^2}\right)$
3	$V_x\left(\frac{C_{ij}-C_{i-1j}}{\Delta x}\right)+V_y\left(\frac{C_{ij+1}-C_{ij-1}}{2\Delta y}\right)=D_x\left(\frac{C_{ij}-2C_{i-1j}+C_{i-2j}}{\Delta x^2}\right)+D_y\left(\frac{C_{ij+1}-2C_{ij}+C_{ij-1}}{\Delta y^2}\right)$
4	$V_x\left(\frac{C_{i+1j}-C_{i-1j}}{2\Delta x}\right)+V_y\left(\frac{C_{ij}-C_{ij-1}}{\Delta y}\right)=D_x\left(\frac{C_{i+1j}-2C_{ij}+C_{i-1j}}{\Delta x^2}\right)+D_y\left(\frac{C_{ij}-2C_{ij-1}+C_{ij-2}}{\Delta y^2}\right)$
5	$V_x\left(\frac{C_{i+1j}-C_{ij}}{\Delta x}\right)+V_y\left(\frac{C_{ij+1}-C_{ij-1}}{2\Delta y}\right)=D_x\left(\frac{C_{i+2j}-2C_{i+1j}+C_{ij}}{\Delta x^2}\right)+D_y\left(\frac{C_{ij+1}-2C_{ij}+C_{ij-1}}{\Delta y^2}\right)$
6	$V_x\left(\frac{C_{i+1j}-C_{ij}}{\Delta x}\right)+V_y\left(\frac{C_{ij+1}-C_{ij}}{\Delta y}\right)=D_x\left(\frac{C_{i+2j}-2C_{i+1j}+C_{ij}}{\Delta x^2}\right)+D_y\left(\frac{C_{ij+2}-2C_{ij+1}+C_{ij}}{\Delta y^2}\right)$
7	$V_x\left(\frac{C_{ij}-C_{i-1j}}{\Delta x}\right)+V_y\left(\frac{C_{ij+1}-C_{ij}}{\Delta y}\right)=D_x\left(\frac{C_{ij}-2C_{i-1j}+C_{i-2j}}{\Delta x^2}\right)+D_y\left(\frac{C_{ij+2}-2C_{ij+1}+C_{ij}}{\Delta y^2}\right)$
8	$V_x\left(\frac{C_{i+1j}-C_{ij}}{\Delta x}\right)+V_y\left(\frac{C_{ij}-C_{ij-1}}{\Delta y}\right)=D_x\left(\frac{C_{i+2j}-2C_{i+1j}+C_{ij}}{\Delta x^2}\right)+D_y\left(\frac{C_{ij}-2C_{ij-1}+C_{ij-2}}{\Delta y^2}\right)$
9	$V_x\left(\frac{C_{ij}-C_{i-1j}}{\Delta x}\right)+V_y\left(\frac{C_{ij}-C_{ij-1}}{\Delta y}\right)=D_x\left(\frac{C_{ij}-2C_{i-1j}+C_{i-2j}}{\Delta x^2}\right)+D_y\left(\frac{C_{ij}-2C_{ij-1}+C_{ij-2}}{\Delta y^2}\right)$

Furthermore, for the convenience sake of computational domain, the following non-dimensional variables are introduced. By taking $\bar{x}=\frac{x}{400}$, $\bar{y}=\frac{y}{320}$, $\overline{V_x}=400V_x$, $\overline{V_y}=320V_y$, $\overline{D_x}=400D_x$, $\overline{D_y}=320D_y$, the physical domain of advection-diffusion equation leads to computation domain $0\le x\le 1$ and also $0\le y\le 1$. The steady state of advection-diffusion equation remains the same after the bar variables are dropped. The domain is being discretised and given the codes. The time-less advection-diffusion equation is solved numerically using the boundary conditions which are obtained from water contents taken on August 28th 2014, at 6 a.m., by [5] as in Table 2. No stability analysis is needed for this steady state problem.

Table 2. Water contents of Unhas lake at three stations

Contents	Station 1 119°29′26.8″E 5°8′13.8″S	Station 2 119°29′26.8″E 5°8′10.2″S	Station 3 119°29′22″E 5°8′6.7″S
Total suspended solid (TSS)	60	22	10
Biological oxygen demand BOD (mg/l)	88	56	48
Chemical oxygen demand COD (mg/l)	220	140	120
Ammonia as N ($NH_3 - N$ mg/l)	1.00	1.85	1.88
Fecal coliform (MPN/100ml)	30	4352	17329

The boundary conditions for these problems as in equation (6) are supplied directly from the water contents from Table 2. Zero boundary conditions are specified anywhere else than the stations' positions.

4. Numerical Results and Discussions

The distribution of coliform bacteria which is using the boundary condition on Station 3 only can be found in Figure 2. Here bacteria concentration is 1.7329 (normalized by dividing with 10000, see Table 2) on top of the boundary. $V_x = V_y = 0$ or no advection involved and $D_x = 6.4 \times 10^{-6}$ and $D_y = 5.1 \times 10^{-6}$. These constants are obtained using assumption that bacteria diffusion coefficient is four times larger than carbon dioxide diffusion coefficient in the water as 0.0016mm^2/s. By dimensionless variables, the above constant numbers are obtained. From computational point of view, this might be useless, since when $V_x = V_y = 0$, the diffusion equation can be multiplied or divided as any number. The only constants that will be taken into account are the ratio between the advection coefficients and the diffusion coefficients.

Figure 2 shows the source of the bacteria begins at Station 3 and then spreading into the lake. This is very different from the results of coliform bacteria obtained from Table 2. However, this suggests that the source of the bacteria does not only come from the top side of the lake, but also comes from the right side of the boundary, as well as any sides of the boundaries.

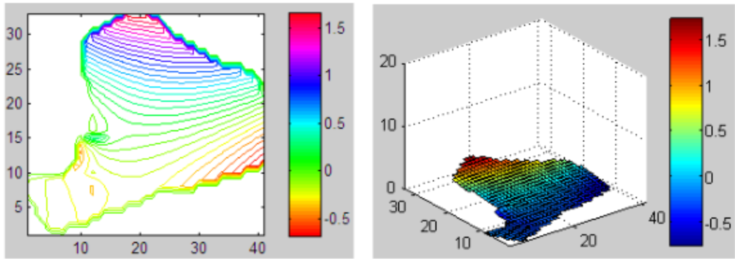

Figure 2. Distribution of fecal coliform using one boundary condition.

Figure 3 shows the distribution of coliform bacteria using three boundary conditions on Stations 1, 2 and 3. Bacteria concentrations are specifying as 0.003, 4.352, 1.7329 on the left, right and top sides of the boundary, respectively, $V_x = V_y = 0$ no advection involved and using normalized values of diffusion coefficients $D_x = 6.4 \times 10^{-6}$ and $D_y = 5.1 \times 10^{-6}$. This means the source of the bacteria not only comes from Station 3 but also comes from Station 2, and no source of bacteria from Station 1.

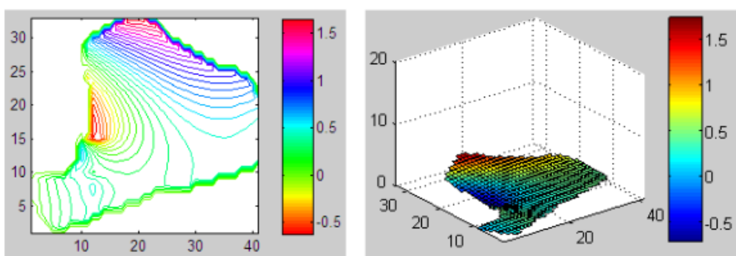

Figure 3. Distribution of fecal coliform using three boundary conditions.

Figure 4 shows the distribution of coliform bacteria using three boundary conditions on Stations 1, 2 and 3. Bacteria concentrations are specifying as 0.003, 0.4352, 1.7329 on the left, right and top sides of the boundary, respectively, with a small advection involved. Here the advection and diffusion constants are $V_x = -1.6 \times 10^{-6}$, $V_y = -1.28 \times 10^{-6}$, $D_x = 1.28 \times 10^{-5}$ and $D_y = 1.02 \times 10^{-5}$. This means the source of the bacteria not only comes from Station 3 but also comes from Station 2, and no source of bacteria from Station 1. From this figure, one can see, even though this still cannot describe a precise distribution, it already describes a better distribution of bacteria concentration of the lake.

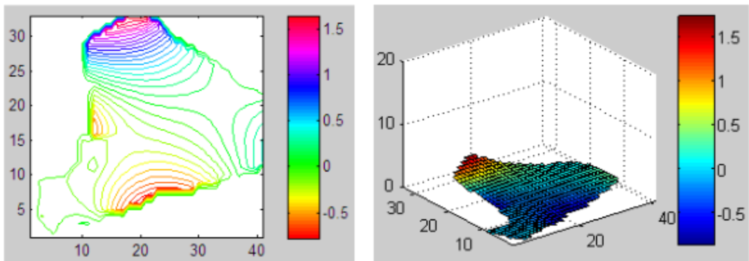

Figure 4. Distribution of fecal coliform using three boundary conditions with advection.

Figure 5 (left) shows the total suspended solid distribution using three boundary conditions and advection diffusion coefficients as $V_x = -1.6 \times 10^{-6}$, $V_y = -12.8 \times 10^{-6}$, $D_x = 1.28 \times 10^{-5}$ and $D_y = 1.02 \times 10^{-5}$. Figure 5 (right) shows the total suspended solid distribution using three boundary conditions and advection diffusion coefficients as $V_x = 1.6 \times 10^{-6}$, $V_y = 1.6 \times 10^{-6}$, $D_x = 1.28 \times 10^{-5}$ and $D_y = 1.02 \times 10^{-5}$. Following figures show that the advection coefficient contributes significantly to the total suspended solid distribution in the lake.

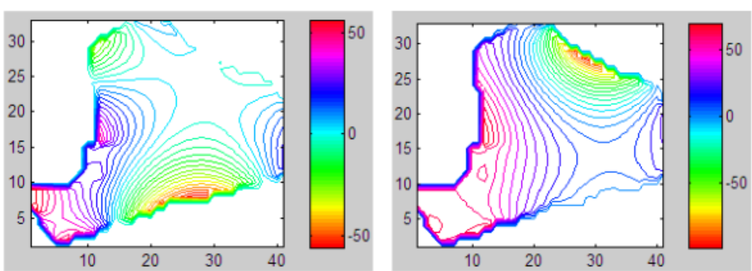

Figure 5. Distribution of total suspended solid.

5. Conclusions

A compound finite difference scheme to the pollution distribution problem may yield significant advantages in monitoring of water quality.

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